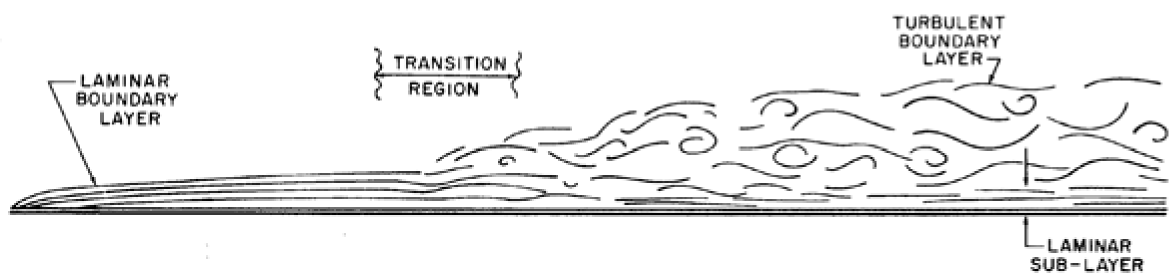
A boundary layer is a thin layer of viscous fluid close to the solid surface of a wall in contact with a moving stream in which the flow velocity varies from zero at the wall (where the flow "sticks" to the wall because of its viscosity) up to the free stream velocity. According to Prandtl this variation in speed takes place in very thin layer. Above this layer, within the free stream the flow is considered to be inviscid. Within the boundary the flow can be either laminar or turbulent. The quality of the boundary layer depend mostly on the Reynolds number, which in case of boundary layers on a flat plate can be defined as:

$$Re_x = \frac{v_\infty x}{\nu},$$

where v_∞ is the free stream velocity, x is the distance from the leading edge of the plate and ν is the kinematic viscosity of the fluid. In case of a flat pate the boundary layer remains laminar if $Re_x < 8.4 \times 10^4$ even in case of strong inlet disturbance, above $Re_x > 8.4 \times 10^4$ the boundary layer can be either laminar or turbulent, however if the Reynolds number exceeds $Re_x \sim > 5 \times 10^5$ the boundary layer turns to be turbulent excluding a very thin laminar *sublayer* just above the solid surface. The laminar-turbulent transition among other factors depends on the turbulence level of the incoming flow, the pressure gradient, the heat transfer and the surface roughness. In case of flow around solid bodies the transition can be delayed with decreasing surface roughness or increasing the flow velocity in the upstream direction like in a confuser.



When the flow within the boundary layer is laminar, the perpendicular distribution to the solid surface of the streamwise velocity can be calculated using the Navier Stokes equation considering the following simplifications:

- The flow within the BL is two dimensional
- The flow velocity component perpendicular to the surface is negligible compared to the streamwise component.
- The gradient of the streamwise velocity in the streamwise direction is negligible compared to the perpendicular to the surface direction.
- The flow is stationary
- The effect of the gravity is neglected
- The static pressure is constant within the BL and it is equal to static pressure in the free stream

Than,

$$\frac{v_x}{v_\infty} \cong 1.5\frac{y}{\delta} - 0.5\left(\frac{y}{\delta}\right)^3,$$

Where y is the distance perpendicular from the surface and δ is the BL thickness and can be estimated using the following equation:

$$\frac{\delta}{x} \cong \frac{5}{\sqrt{Re_x}} \ .$$

In case of turbulent boundary layers the above mentioned properties can be estimated using Pandtl's mixing length mixing length model and the following approximations:

At the wall the mixing length is zero and increasing linearly in the perpendicular direction: $l \cong \kappa y$

The shear stress in the near wall region is constant and it is equal to the τ_0 wall shear stress.

At the wall $\frac{\partial v_x}{\partial y} > 0$

Than:

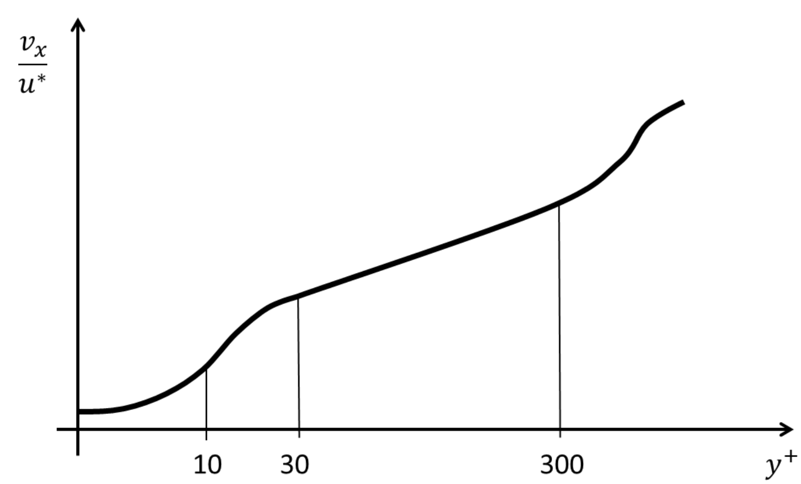
$$\frac{v_x}{u^*} = \frac{1}{\kappa} ln \frac{yu^*}{\nu} + K$$

Where $u^* = \sqrt{\frac{\tau_0}{\rho}}$ is the so called friction velocity and K and κ are constants. This equation is the universal (or logarithmic) law of wall. Based on experimental data the value of κ (von Kármán constant) is 0.41 and K=5.

In the vicinity of the wall the solid surface restrict the development of any vortex structures. This part of the BL is the laminar sublayer. Here the velocity distribution is described by the following equation:

$$\frac{v_x}{u^*} = \frac{yu^*}{\nu}.$$

Introducing the $y^+ = \frac{yu^*}{\nu}$ dimensionless wall distance the laminar sublayer lays within the $y^+ < 10$ region, the universal law of wall is valid between $30 < y^+ < 300$. Some empirical or semi empirical correlation could be used to estimate the velocity distribution in the outer $300 \ < y^+$region.



In case of a flat plate the thickness of the BL can be estimated using the following equation:

$$\frac{\delta}{x} \cong \frac{0.14}{\sqrt[7]{Re_x}}$$

The purpose of the measurement:

Here we will investigate the development of the BL within a rectangular channel using a simple Pitot probe. The BL within this channel is slightly different from the theoretical BL on a flat plate. Since the intake is not ideal the flow can detach from the sharp edges, and the thickening BL increases the free cross section therefore increasing the speed in the free stream hence also stabilizing the flow within the BL delaying the laminar turbulent transition. Therefore the validity of the above equations has to be investigated in the given environment.

Estimating the constants for the universal BL equation:

The von Kármán constant is taken to be $\kappa = 0.41$. To calculate the friction velocity the friction stress on the walls is taken to be uniform which than can be estimated from the streamwise pressure drop for a given section:

$$\tau_0 = \frac{A}{K_n}\frac{\partial p}{\partial x} \cong \frac{A}{K_n}\frac{\Delta p}{L}$$

Where A is the area of the rectangular cross section, K_n is the wetted surface of the cross section, Δp is the pressure drop and L is the length of the given section.

Determining the thickness of the BL

The δ BL thickness is defined by the point where the velocity of the flow reaches 99% of the free stream velocity, i. e. x where $v_x \geq 0.99 v_\infty$.

Displacement thickness:

The displacement thickness, δ_1 is the distance by which a surface would have to be moved in the direction perpendicular to its normal vector away from the reference plane in an inviscid fluid stream of velocity v_∞ to give the same flow rate as occurs between the surface and the reference plane in a real fluid.

$$\delta_1 = \frac{1}{v_\infty}\int_0^\infty (v_\infty - v_x)dy \cong \frac{1}{v_\infty}\int_0^{\delta_1} (v_\infty - v_x)dy$$

Momentum thickness:

The momentum thickness, δ_2, is the distance by which a surface would have to be moved parallel to itself towards the reference plane in an inviscid fluid stream of velocity v_∞ to give the same total momentum as exists between the surface and the reference plane in a real fluid

$$\delta_2 = \int_0^\infty \frac{v_x}{v_\infty}\left(1 - \frac{v_x}{v_\infty}\right) dy \cong \int_0^{\delta_2} \frac{v_x}{v_\infty}\left(1 - \frac{v_x}{v_\infty}\right) dy$$

Shape factor:

A shape factor is used in boundary layer flow to determine the nature of the flow:

$$H = \frac{\delta_1}{\delta_2}$$

The higher the value of H, the stronger the adverse pressure gradient. A high adverse pressure gradient can greatly reduce the Reynolds number at which transition into turbulence may occur. Conventionally, H = 2.59 (Blasius boundary layer) is typical of laminar flows, while H = 1.3 – 1.4 is typical of turbulent flows.